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In-House Report
September 1984

NULL SYNTHESIS WITH PHASE AND AMPLITUDE CONTROLS AT THE SUBARRAY OUTPUTS

Randy L. Haupt, Captain, USAF

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, NY 13441

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APPROVED: *Philipp Blacksmith*

PHILIPP BLACKSMITH
Chief, EM Techniques Branch
Electromagnetic Sciences Division

APPROVED: *Allan C. Schell*

ALLAN C. SCHELL
Chief, Electromagnetic Sciences Division

FOR THE COMMANDER: *John A. Ritz*

JOHN A. RITZ
Acting Chief, Plans Office

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This report investigates the generation of nulls in an antenna pattern by controlling the phase and amplitude of the subarray output signals. Although the nulls may be formed with any number of subarrays, the subarray configuration and desired null location have a major impact on the distortion in the far-field pattern that results. The distortion problem resulting from subarray nulling is well known, but little theoretical analysis has been done to explain the cause of the distortion. This report offers a theoretical and graphical explanation of why the distortion occurs.

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Randy L. Haupt	(617) 861-2059	EECS

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I. INTRODUCTION

As the microwave spectrum becomes more and more crowded with users,
interference rejection techniques become increasingly necessary. One way to re-
duce the interference is to generate a null in the antenna pattern sidelobes in the
direction of the interference. It is possible to form these nulls for a phased array
antenna by adjusting the phase and amplitude of the signals received at each element.
Large arrays require expensive hardware or a lot of time to form the nulls. In
contrast, the nulls can be formed quickly if there are receivers or correlators at
every element. Normally, this equipment is not part of the antenna and must be
add. d at considerable cost. On the other hand, forming nulls by "searching" for
the best phase and amplitude settings avoids the expensive equipment, but takes
considerable time to form the nulls.

One way to reduce either the amount of extra hardware or the time to form the
null is to control the signal characteristics at the subarray output rather than at
the individual element outputs. [1,2] Often a large array is divided into subarrays

1. Chapman, D. J. (1976) Partial adaptivity for the large array, IEEE Trans.
   Antennas Propag., AP-24:685-696.
2. Morgan, D. R. (1978) Partially adaptive array techniques, IEEE Trans.
   Antennas Propag. AP-26:823-833.

1

in order to place time delay units at the subarray outputs. Arrays sometimes need time delay units to receive wide bandwidth signals properly. Time delay units are very expensive and bulky so they are usually not placed at every element in the array. Thus, only the subarray outputs receive a true time delay. Subarray outputs also offer convenient locations to put controls that modify the signals to generate nulls in the far-field pattern. Fewer signals to control implies either less hardware or less time needed to form nulls. These advantages make sub-array nulling very attractive.

The problem of antenna pattern distortion due to subarray nulling is well known. Reducing the number of signal controls in the array in turn reduces the amount control over the far-field pattern. This report illustrates why that distortion occurs.

2. NULL SYNTHESIS

This section explores the limitations to forming nulls with phase and amplitude controls at the subarray outputs. Figure 1 is a linear array divided into M contiguous subarrays of N elements per subarray. Each array element has a beam-steering phase shifter. For the purpose of this analysis, the phase shifters are assumed to be set for maximum gain at boresite. The amplitude weights correspond to an amplitude taper, such as a Taylor distribution. In addition, a phase shifter and time delay unit appear at the output of each subarray. Since the main beam is assumed to be at boresite, then all the time delay units are set at zero and can be ignored in the analysis.

Equation (1) gives the far-field pattern of a linear array of M subarrays and N elements per subarray.

$$F(u) = \sum_{m=1}^{M} (1 + \alpha_m + j\beta_m) \sum_{n=1}^{N} a_{mn} \exp(jk\, d_{mn}\, u) \tag{1}$$

where

$$1 + \alpha_m + j\beta_m = \text{adjustable complex weight at subarray m}$$
$$a_{mn} = \text{amplitude weight at element n of subarray m}$$
$$k = \text{wave number} = 2\pi/\lambda$$
$$\lambda = \text{wavelength}$$
$$d_{mn} = \text{distance in } \lambda \text{ from center of array to element n in subarray m}$$
$$u = \sin\theta$$
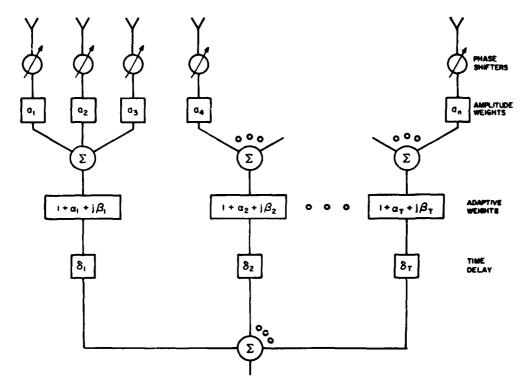$$\theta = \text{direction from boresite}$$

2

Figure 1. Linear Array Divided Into Subarrays

Ideally, the antenna forms a null in the direction of interference. In other words, $F(u_q) = 0$ when u_q is in the direction of an interference source and $q = 1, 2, \ldots Q$. The nulls appear in the desired direction when the complex weights at the subarray outputs (written here in real and imaginary form) are set at the proper values.

If nulls form in Q desired directions then[3]

$$\sum_{m=1}^{M} (1 + \alpha_m + j\beta_m) \sum_{n=1}^{N} a_{mn} \exp(jk\, d_{mn}\, u_q) = 0; \quad q = 1, 2, \ldots Q \qquad (2)$$

$$\sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn} \exp(jk\, d_{mn}\, u_q) + \sum_{m=1}^{M} (\alpha_m + j\beta_m) \sum_{n=1}^{N} a_{mn} \exp(jk\, d_{mn}\, u_q) = 0$$

$$(3)$$

3. Haupt, R. L. (1983) Nulling With Limited Degrees of Freedom, RADC-TR-83-114, AD A132276.

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$$\sum_{m=1}^{M} (\alpha_m + j\beta_m) \sum_{n=1}^{N} a_{mn} \exp(jk\, d_{mn}\, u_q) = - \sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn} \exp(jk\, d_{mn}\, u_q) \,.$$

(4)

Equation (4) represents a set of equations with M unknowns. Consequently, the equation may be written in the matrix form $\underset{\sim}{A}\underset{\sim}{X} = \underset{\sim}{B}$ where

$$\underset{\sim}{A} = \begin{bmatrix} \sum_{n=1}^{N} a_{1n} \exp(jk\, d_{1n}\, u_1) & \cdots & \sum_{n=1}^{N} a_{Mn} \exp(jk\, d_{Mn}\, u_1) \\ \vdots & \vdots & \vdots \\ \sum_{n=1}^{N} a_{1n} \exp(jk\, d_{1n}\, u_Q) & & \sum_{n=1}^{N} a_{Mn} \exp(jk\, d_{Mn}\, u_Q) \end{bmatrix}$$

$$\underset{\sim}{X} = \begin{bmatrix} \alpha_1 + j\beta_1 \\ \vdots \\ \alpha_m + j\beta_m \end{bmatrix}$$

$$\underset{\sim}{B} = \begin{bmatrix} -\sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn} \exp(jk\, d_{mn}\, u_1) \\ \vdots \\ -\sum_{m=1}^{M} \sum_{n=1}^{N} a_{mn} \exp(jk\, d_{mn}\, u_Q) \end{bmatrix}$$

$\underset{\sim}{X}$ has a unique solution when $Q = M$. Usually, though, $Q < M$ and several different values of X satisfy the equation $\underset{\sim}{A}\underset{\sim}{X} = \underset{\sim}{B}$. Small values for the complex weights will disturb the far-field pattern less than larger values. Thus, of the many possible values for $\underset{\sim}{X}$, the best results occur from minimizing $\Sigma(\alpha^2 + \beta^2)$. Solving $\underset{\sim}{A}\underset{\sim}{X} = \underset{\sim}{B}$ while minimizing the complex weights gives[4]

$$\underset{\sim}{X} = \underset{\sim}{A}^{\dagger} (\underset{\sim}{A}\underset{\sim}{A}^{\dagger})^{-1} \underset{\sim}{B}$$

(5)

---

4. Shore, R.A., and Steyskal, Hans (1982) Nulling in Linear Array Patterns With Minimization of Weight Perturbations, RADC-TR-82-32, AD A118695.

where $\underset{\sim}{A}^{\dagger}$ is the transpose conjugate of matrix $\underset{\sim}{A}$. Solving Eq. (4) in matrix form form gives the following values for the elements in the complex matrix $\underset{\sim}{X}$:

$$\alpha_m = \sum_{q=1}^{Q} \sum_{n=1}^{N} a_{mn} [Y_q \cos(k\,d_{mn}\,u_q) + Z_q \sin(k\,d_{mn}\,u_q)] \tag{6}$$

$$\beta_m = \sum_{q=1}^{Q} \sum_{n=1}^{N} a_{mn} [Z_q \cos(k\,d_{mn}\,u_q) - Y_q \sin(k\,d_{mn}\,u_q)]\ . \tag{7}$$

The variables Y_q and Z_q are elements of a complex array W given by

$$\underset{\sim}{W} = (\underset{\sim}{A}\underset{\sim}{A}^{\dagger})^{-1}\ \underset{\sim}{B}. \tag{8}$$

Figure 2 shows the quiescent far-field pattern of a 24-element array of isotropic sources spaced 0.5 λ apart and having a uniform amplitude distribution. Figures 3a, 4a, and 5a show the results of nulling with an interference source at 8° and with 24, 8, and 4 subarrays, respectively. As the number of subarrays decrease, the distortion to the antenna pattern increases. The increase in distortion is not significant, though.
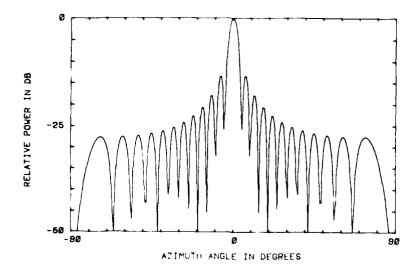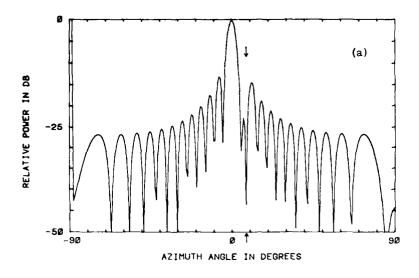
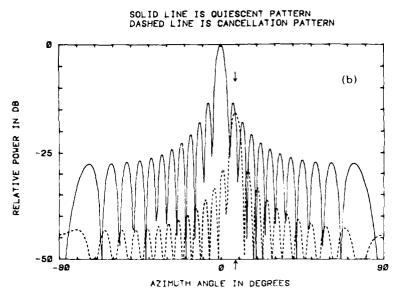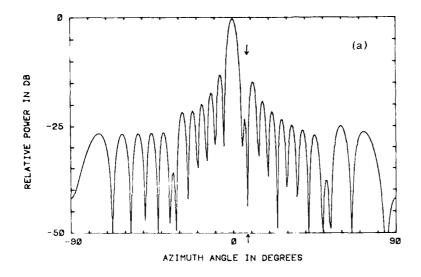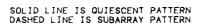Figure 2. Quiescent Far-Field Pattern of a Twenty-Four Element Uniform Array

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Figure 3. Placing a Null at 8° With Twenty-Four Subarrays.
(a) Far-Field Pattern, (b) Cancellation Beam Superimposed
on Quiescent Pattern

Problems with pattern distortion occur when the null is formed outside of the
subarray mainbeam pattern. A subarray pattern is the far-field pattern of an individual subarray with its peak centered at 0°. Although the amplitude tapers for each
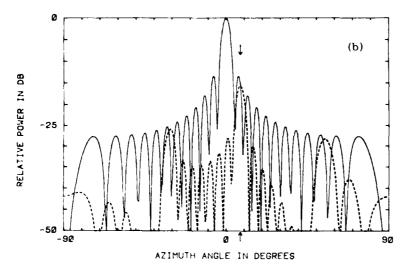
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Figure 4. Placing a Null at 8° With Eight Subarrays. (a) Far-Field Pattern, (b) Cancellation Beam Superimposed on Quiescent Pattern

sub ray are different, their beamwidths are about equal because they are all the same size. The null-to-null beamwidth BW of a uniformly illuminated array with N elements spaced 0.5 λ apart is given by

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At $\pm \theta_{NULL}$ the peak of the cancellation beam is the same height as one of the first grating lobes. Beyond this angle the grating lobe gain becomes larger than that of the mainbeam. Figure 14b is an example where the cancellation beam main lobe and first grating lobe gains are nearly equal. They would be the same level at $\theta_{NULL} = 20.9°$.

One final point worth mentioning is that the distortion would be greater in the neighborhood of a null in the subarray pattern. The amount of distortion would improve when the null was placed in the same location as the peak of a subarray pattern sidelobe. For instance, a null placed at 38° with eight subarrays produces considerably more distortion than a null placed at 60°. The null at 38° is close to the null in the subarray pattern while 60° is almost at the peak of the subarray pattern sidelobe.

4. CONCLUSIONS

This report illustrates theoretically the pattern distortion problem associated with subarray nulling. The amount of distortion to the far-field antenna pattern is inversely proportional to the gain of the subarray far-field pattern. Thus, subarray nulling near the mainbeam produces little distortion. On the other hand, the distortion increases dramatically when the null is placed further from the mainbeam.

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The relative levels of the subarray pattern at the array factor mainbeam location and some grating lobe locations are given in Table 1. Also the relative levels of the cancellation beam at these locations are given. The subarray pattern and the cancellation beam have the same relative difference between them at the locations listed. This proportional difference results from the product of the array factor and the subarray pattern that leads to the cancellation beam.

Table 1. Comparison of the Relative Power Level of the Cancellation Patterns and Subarray Patterns
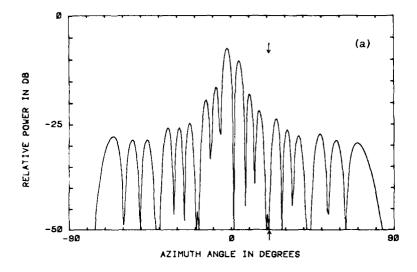
Null Location	Number of Subarrays	Mainbeam and Grating Lobe Locations	Relative Levels* of Cancellation Pattern	Relative Levels* of Subarray Pattern	Associated Figures
8°	8	8°	-15.6 dB	-18.6 dB	7, 4b
		33.8°	-28.4	-31.4	
21°	8	21°	-24	-22.2	7, 10b
		20.8°	-23.9	-22.1	
38°	8	38°	-26.5	-38.4	7, 14b
		-3.8°	-46.3	-18.2	
8°	4	8°	-15.6	-14.6	8, 5b
		-11.5°	-18.8	-17.8	
		-33.8°	-27.6	-26.6	
21°	4	21°	-24	-34.8	8, 11b
		1.5°	-1.3	-12.1	
		-20.8°	-25.1	-35.9	
38°	4	38°	-26.5	-32.6	8, 15b
		18.5°	-31.5	-37.6	
		-3.8°	-6.5	-12.6	

*in dB relative to the peak gain of a uniform array of 24 elements.

The relative levels of the subarray pattern and cancellation pattern at the array factor mainbeam and grating lobe locations gives an idea of how much the antenna pattern will be distorted due to nulling. Look at the figures associated with the entries in Table 1 and see how the far-field pattern distortion relates to the subarray pattern. The lower the level of the subarray pattern in the direction of the desired null, the greater the pattern distortion becomes when the null is formed.

A conservative estimate of the angular limit of subarray nulling is given in Eq. (11)

$$\theta_{NULL} = \pm 0.5 \sin^{-1} [1/(Nd)] . \tag{11}$$

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SOLID LINE IS QUIESCENT PATTERN
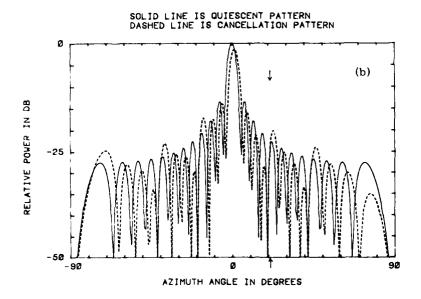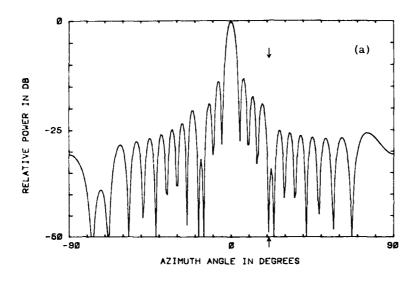DASHED LINE IS CANCELLATION PATTERN

Figure 15. Placing a Null at 21° With Four Subarrays.
(a) Far-Field Pattern, (b) Cancellation Beam Superimposed
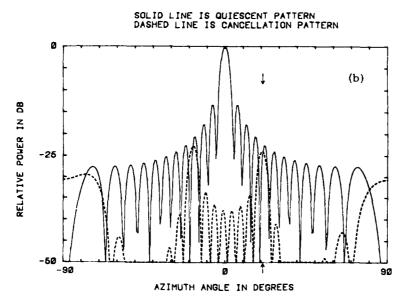on Quiescent Pattern

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Figure 14. Placing a Null at 21° With Eight Subarrays.
(a) Far-Field Pattern, (b) Cancellation Beam Superimposed
on Quiescent Pattern

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The problem of pattern distortion associated with subarray nulling is due to the grating lobes in the array factor. As the array factor's mainbeam moves away from boresite, the product of the array factor's mainbeam and subarray pattern decreases. In addition, the first grating lobe begins to move inside the subarray pattern's mainbeam. Consequently, the peak of the cancellation beam reduces in size while the grating lobe becomes larger in size. Adding the cancellation pattern to the quiescent pattern creates a null in the direction of the cancellation pattern's main beam, but causes distortion to the pattern in the direction of the grating lobe.

Consider what happens when a null is placed in the antenna pattern with eight subarrays. The three-element subarray pattern is shown in Figure 7. From the above discussion, one would expect little distortion to the antenna pattern when nulling at the peak of a sidelobe between 9° and 15°, because the cancellation pattern's grating lobe is less than the cancellation pattern's main beam. Between 15° and 30°, the cancellation pattern's first grating lobe grows and gradually becomes larger than the cancellation pattern's main lobe. From 30° to 40°, the cancellation pattern's grating lobe is significantly higher than its mainbeam.

Figures 4a and 4b show the results of placing a null at 8°, Figures 14a and 14b result from a null at 21°, and Figures 10a and 10b result from a null at 38°. Note how the cancellation pattern's mainbeam grows smaller as its grating lobe grows larger when the null moves further away from boresite. This increase in the cancellation pattern's grating lobe corresponds to the increase in distortion to the nulled pattern.

A similar analysis is possible with the four subarray example, except the amount and location of the distortion changes. Since the subarray pattern (Figure 7) has a much narrower main beam, distortion becomes more of a problem as the null moves away from boresite. This fact is evident in Figures 5a, 15a, and 11a. The corresponding (b) parts of these figures show the change in the cancellation beam's main lobe and grating lobe.

The amount of distortion to the antenna pattern may be estimated from the location of the grating lobes, the subarray pattern, and the height of the quiescent sidelobe before nulling. Below is a list of the grating lobe locations for the 8-element array factor and 4-element array factor. These were calculated from Eq. (10) and can be seen in Figures 12 and 13.

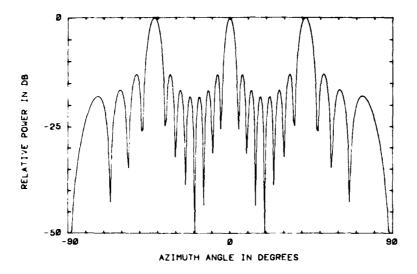8 elements - ±41.8°

4 elements - ±19.5°, ±41.8°, ±90° .

Figure 12. Array Factor for a Three-Element Array With
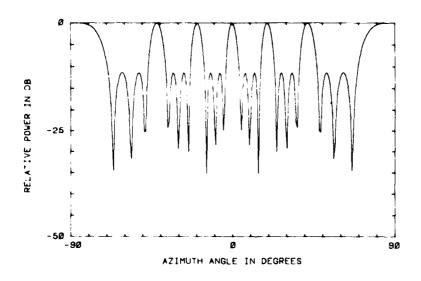a Spacing of 1.5 λ

Figure 13. Array Factor for a Six-Element Array With a
Spacing of 3.0 λ

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3. CAUSES OF FAR FIELD PATTERN DISTORTION

Far field pattern distortion caused by subarray nulling limits its use. The limitations are fundamental and do not depend on the technique that generates the nulls. This limitation is best illustrated by examining the cancellation beam that adds to the quiescent pattern to form the null. Equation (3) is written in the form

QUIESCENT PATTERN + CANCELLATION PATTERN = 0

at the desired null location. The nulling technique forms a cancellation pattern that has the same amplitude, but is 180° out of phase with the quiescent pattern at the interference location. Adding the two patterns together produces a null in the direction of the interference.

The cancellation beams that produce the adapted patterns in part (a) of Figures 3, 4, 5, 9, 10, and 11 are shown in part (b) of those same figures. When the array is fully adaptive (24 subarrays), the cancellation beam has one peak and no grating lobes. Grating lobes do not enter real space in the array factor because the elements are spaced 0.5λ apart. Since the element pattern is isotropic, the cancellation pattern and array factor are identical.
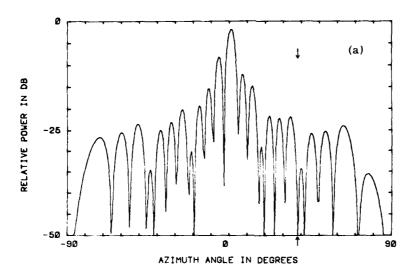
When the array is divided into subarrays, it may be considered as an array of M elements spaced $N\lambda/2$ apart. Each array element, or subarray, now consists of N isotropic antennas. The subarray patterns appear in Figures 7 and 8. Adjusting the phase and amplitude of the signals at the subarray outputs does not change the subarray patterns; however, controlling the subarray output signals can modify the array factor. Figures 12 and 13 show the quiescent array factors for M = 8 and 4, respectively. The array factors have grating lobes because of the large element (subarray) spacing. Grating lobes appear at the angles

$$\theta = \pm \sin^{-1}|x/Nd] \qquad (10)$$

where

$$x = 1, 2, 3$$

$$d = \text{element spacing} .$$

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SOLID LINE IS QUIESCENT PATTERN
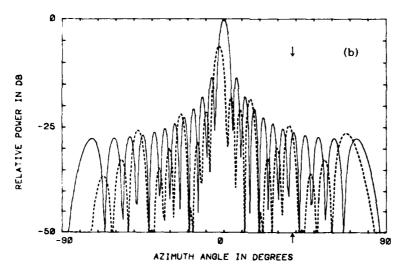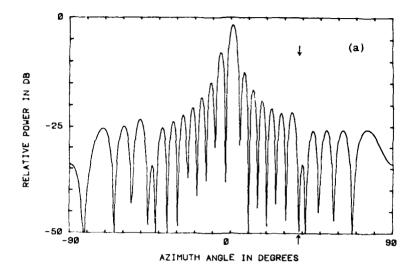DASHED LINE IS CANCELLATION PATTERN

Figure 11.   Placing a Null at 38° With Four Subarrays.
(a) Far-Field Pattern,   (b) Cancellation Beam Superimposed
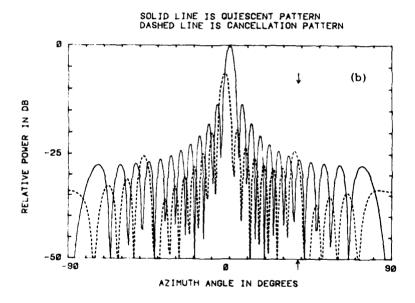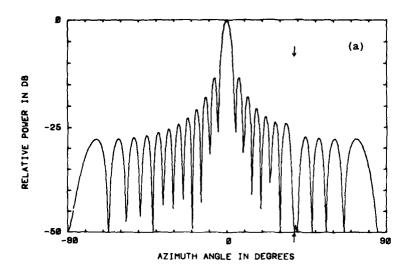on Quiescent Pattern

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Figure 10. Placing a Null at 38° With Eight Subarrays.
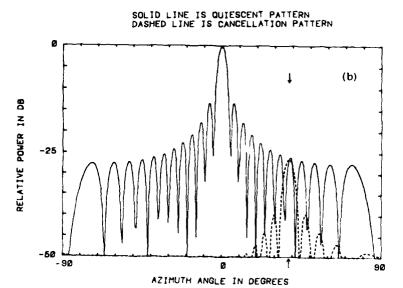(a) Far-Field Pattern, (b) Cancellation Beam Superimposed
on Quiescent Pattern

12

Figure 9. Placing a Null at 38° With Twenty-Four Subarrays.
(a) Far-Field Pattern, (b) Cancellation Beam Superimposed
on Quiescent Pattern

11

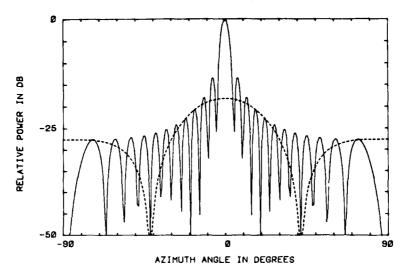SOLID LINE IS QUIESCENT PATTERN
DASHED LINE IS CANCELLATION PATTERN

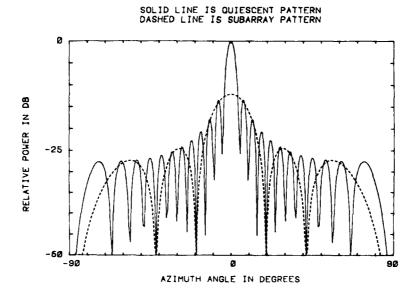Figure 7. Far-Field Pattern of a Three-Element Subarray Superimposed on the Quiescent Pattern

SOLID LINE IS QUIESCENT PATTERN
DASHED LINE IS SUBARRAY PATTERN

Figure 8. Far-Field Pattern of a Six-Element Subarray Superimposed on the Quiescent Pattern

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equally in all directions. Figures 6, 7, and 8 show the subarray patterns super-
imposed on the quies ~nt pattern for N = 1, 3, and 6 elements. If our initial
premise of nulling inside the subarray pattern produces less distortion than nulling
outside the subarray pattern is true, then an interference source at 38° should
produce more distortion than one at 8°. The distortion should not change for the
24 subarray case because the subarray pattern is isotropic (see Figure 6). How-
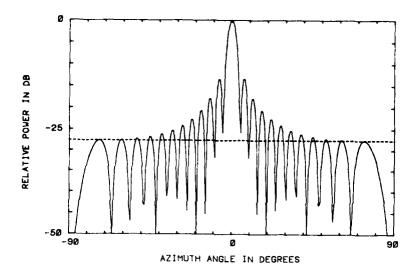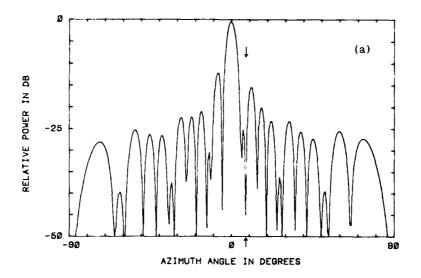ever, the 8 and 4 subarrays cases should show a marked degradation.

Figure 6. Far-Field Pattern of an Isotropic Element
Superimposed on the Quiescent Pattern

Figures 9a, 10a, and 11a are the resulting patterns after placing a null at 38°.
As predicted, when there were 24 subarrays, the amount of distortion did not
change when the interference location moved. Because the interference location
moved outside or nearly outside the subarray pattern, the amount of distortion to
the far-field pattern increased substantially. The next section offers an explanation
of this distortion phenomenon.

9

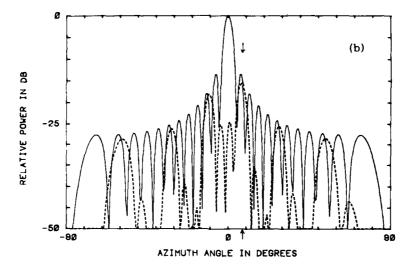SOLID LINE IS QUIESCENT PATTERN
DASHED LINE IS CANCELLATION PATTERN

Figure 5. Placing a Null at 8° With Four Subarrays. (a) Far-
Field Pattern, (b) Cancellation Beam Superimposed on
Quiescent Pattern

$$BW = 2 \sin^{-1}(2/N) . \qquad (9)$$

The null-to-null beamwidths for three and six elements are 84° and 39° re-
spectively. An isotropic element has no null-to-null beamwidth because it radiates

8

END

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